A NEW PROBABILISTIC METHOD FOR FORECASTING MORTALITY: A CASE STUDY OF ESTONIA

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Abstract. A new probabilistic mortality forecasting approach is introduced that unlike the Lee-Carter Method (and its variants) is directly linked to the fundamental demographic equation, the cornerstone of demographic theory. This is an important consideration in developing accurate forecasts. Because it forecasts "years lived," this new approach directly yields life expectancy and a corresponding future life table, which is not the case with the Lee-Carter Method and its variants. In an ex post facto evaluation using Estonian data from the Human Mortality Database, the new approach was found to provide accurate forecasts of "years lived by age" (${}_nL_x$) both in terms of point and interval measures over a 20 year period. Probabilistic ${}_nL_x$ forecasts for Estonia are then provided, the results are discussed, and the next steps in evaluating this approach are suggested.

Background. Mortality Forecasting is an important activity. It is used in the preparation of population forecasts based on the cohort component (CCM) method (Smith, Tayman, and Swanson, 2013: 61-72), the development of social welfare, annuity and pension products (Lee and Miller, 2001; Booth and Tickle, 2008; Haberman and Renshaw, 2011; Huang, Maller, and Ning, 2020; Rabbi and Mazzuco, 2020; Shang, Booth, and Hyndman, 2011; Tabeau, Van Den Berg Jeths, and Heathcote, 2001) and epidemiological/health research (Andrade, Camarda, and Arolas, 2025; Swanson, Bryan, and Chow, 2020; Booth and Tickle, 2008).

The Lee-Carter approach to forecasting mortality was introduced in 1992 (Lee and Carter, 1992) and along with its refinements and variants is arguably the most widely used approach in the world (Booth and Tickle, 2008; Rabbi and Mazzuco, 2020; Basellini, Camarda, and Booth, 2023). As observed by Basellini, Camarda, and Booth (2023: 1034), it is useful to examine the Lee-Carter approach in terms of two aspects, the model and the method. The model is a functional form for age-specific mortality (age-specific death rates, ASDRs) and the method consists of a series of steps to estimate the model and fit a time series model to the time index, along with specific adjustment and estimation procedures. Both the model and the method are essentially mathematical fitting procedures and have no direct relationship to the dynamics of population change and the components of those change, one of which is obviously mortality. Moreover, in order to generate a forecast of life expectancy and a corresponding life table, the Lee-Carter

Method and its variants require that the ASDRs be turned into a life table (e.g., Fergany's method (Fergany, 1971) and the Keyfitz-Frauenthal method (Kintner, 2004: 314-315)).

This paper introduces a new method of mortality forecasting by showing how measures of uncertainty from a standard time series model, "Auto Regressive Integrated Moving Average" ("ARIMA," Box and Jenkins, 1976), can be applied to a population projection based on the Hamilton-Perry Method ("H-P," Baker et al., 2017) that generates "years lived" by age ($_nL_x$) and which also includes Total years lived (T_0) and life expectancy at birth (e_0) as found in an abridged period life table. From the perspective of formal demography, this is a forecast of the age distribution and size of the stationary population associated with the mortality and the age structure of a given population (see, e.g., Ryder, 1975; Rao and Carey, 2015; Swanson and Tedrow, 2021).

The measures of forecast uncertainty are relatively easy to calculate and meet several important criteria used by demographers who routinely generate forecasts, including utility (Tayman and Swanson, 1996) as well as face validity, plausibility, production cost, timeliness, ease of application and ease of explanation (McNown, Rogers, and Little, 1995; Smith, Tayman, and Swanson, 2013: 302-315). Unlike the Lee-Carter method and its corresponding "principal components" variants (Booth and Tickle, 2008; Lee and Carter, 1992; Lee and Miller, 2001; Shang, Booth, and Hyndman, 2011) this approach, given three major constraints (described later), links the probabilistic forecast uncertainty to the fundamental demographic equation, the cornerstone of demographic theory. In addition to being a potential contribution to formal demography, this is an important consideration in developing accurate forecasts (Swanson, et al. 2023). Also, unlike the Lee-Carter method and its variants, this new approach directly yields life expectancy and a corresponding future life table because it directly forecasts "years lived" by age ($_nL_x$). An ex post facto evaluation of the accuracy of the method is conducted in the form of a case study using

Estonian data found at the Human Mortality Data Base (HMD) and an example set of forecasts using current data is provided for Estonia.

Data. We selected Estonia for this case study mainly for two reasons. First, its population is small: As of 1 January 2025, Statistics Estonia (2025) shows it as 1,369,995. We wanted a small population in this case study because our experience in working with large and small populations suggested that if the evaluation of our proposed method shows that it works well in a small population, it is, with some caveats (found in the "Evaluation" and "Discussion" sections), likely to work not only work in other small populations but also in large populations. Second, its data are of high quality and found in HMD (2025), which is where we obtained annual ${}_nL_x$ data from 1959 to 2024 that were organized in such a manner that made it easy to assemble into 18 age groups (0-4, 5-9, 10-14,…,75-79, 80-84, and 85+). We also computed the ratio ${}_nLx/42,388$, where 42,288 is the land area of Estonia (km^2) so that we had annual nLx "density" values for each of the 18 age groups from 1959 to 2024.We discuss why we computed these density values in the "Transferring Uncertainty" section.

Method. We employ the Hamilton-Perry (H-P) method (Baker et al., 2017), which computes cohort change ratios (CCRs) using two counts of the age-structure (${}_nL_x$) in question, typically five or ten years apart, which directly capture age-specific population dynamics. Before turning to a discussion of the probabilistic approach we use (which is followed by a description of our input data and the projection results), it is helpful to note that the H-P method is algebraically equivalent to the fundamental demographic equation and therefore grounded in demographic theory (Baker et al., 2017: 251-252),. Barring unforeseeable catastrophes and other events that have very low probabilities of occurring (Taleb, 2010), as noted earlier, the closer one comes to having accurate data embedded in a method that is grounded in demographic theory, the more accurate a population projection method will likely be (Swanson et al., 2023), a dictum that one could reasonably expect to apply to forecasting ${}_nL_x$.

There are three *components of change* in a population: mortality, fertility, and migration. The overall growth or decline of a population is determined by the interplay among these three components. The exact nature of this interplay can be formalized in the *fundamental demographic equation*:

$$P_l - P_b = B - D + IM - OM, \qquad [1]$$

Where P_l is the population at the end of the time period; P_b is the population at the beginning of the time period; and B, D, IM, and OM are the number of births, deaths, in-migrants, and out-migrants during the time period, respectively. The difference between the number of births and the number of deaths is called *natural change* (B – D); it represents population growth coming from within the population itself. It may be either positive or negative, depending on whether births exceed deaths or deaths exceed births. The difference between the number of in-migrants and the number of out-migrants is called *net migration* (IM – OM); it represents population growth coming from the movement of people into and out of the area. It may be either positive or negative, depending on whether in-migrants exceed out-migrants or out-migrants exceed in-migrants. In cases where IM and OM do not occur (e.g., the world as a whole, the stationary population that is found in a life table), these elements can be omitted from the fundamental population equation.

The fundamental demographic equation can also be extended to apply to age groups, age-gender groups, and age-gender-race groups, as well as age-gender-ethnicity groups. This type of extension forms the logical basis of the equation and can be used to project a population into the future by age, age and gender, or by age, gender, and race. Once launched, these components (which are frequently modified as the projection moves into the future based on assumptions about their direction) are applied to the resulting age-gender structure at each cycle of the projection. In terms of ${}_nL_x$ there is no migration, which eliminates the need for this component in forecasting ${}_nL_x$.

The Hamilton-Perry Method of Population Projection. The Hamilton-Perry (H-P) method (Baker et al., 2017: 251-252) conforms to the fundamental population equation but it does not apply the separate components of population change to the age structure at the launch year. Instead, it computes cohort change ratios (CCRs) using two counts of the age-structure in question, typically five or ten years apart, which directly capture mortality and migration. The fertility component uses a "child-adult ratio" from the most recent age structure data or a "child-woman ratio" for a projection by gender. It is well-suited for generating a population projection, as well as ${}_nL_x$, per the framework found in Swanson et al. (2023): (1) It corresponds to the dynamics by which a population moves forward in time; (2) there is information available relevant to these dynamics; (3) the time and resources needed to assemble relevant information and generate a projection are minimal; and (4) the information needed from the projection is generated by the H-P method.

The H-P method moves a population by age (and gender) from time t to time t+k (the projection cycle length) using CCRs computed from data in the two most recent data points (e.g., censuses or estimates) with the proviso that the width of the age groups (other than the terminal, open-ended age group) can be divided into the length of the projection cycle such that it yields a whole number as the quotient. It consists of two steps. The first uses existing data to develop CCRs, and the second applies the CCRs to the cohorts of the launch year population to move them into the future. The formula for the first step, the development of a CCR, is:

$${}_nCCR_{x,i} = {}_nP_{x,i,t} / {}_nP_{x-k,i,t-k}, \qquad [2]$$

where

${}_{n}P_{x,i,t}$ is the population aged x to x+n in area i at the most recent census/estimate (t),

${}_{n}P_{x-k,i,t-k}$ is the population aged x-k to x-k+n in area i at the 2nd most recent census/estimate (t-k),

k is the number of years between the most recent census/estimate at time t for area i and the census/estimate preceding it for area i at time t-k.

The basic formula for the second step, moving the cohorts of a population into the future, is:

$${}_{n}P_{x+k,i,t+k} = ({}_{n}CCR_{x,i}) \times ({}_{n}P_{x,i,t}), \qquad [3]$$

where

${}_{n}P_{x+k,i,t+k}$ is the population aged x+k to x+k+n in area i at time t+k

Given the nature of the CCRs, they cannot be calculated for the youngest age group (i.e., ages 0-4 if it is a five-year projection cycle; ages 0-9 if it is a ten-year projection cycle), because this cohort came into existence after the census/estimate data collected at time t-k. To project the youngest age group, one uses the “Child-Adult Ratio” (CAR), where the number in the youngest age group at time t is divided by the number of adults at time t who are of childbearing age (e.g., 15-44). It does not require any data beyond what is available in the census/estimate sets of successive data.

The CAR equation for projecting the population aged 0-4 is:

$$\text{Population 0-4: } {}_{5}P_{0,t+k} = ({}_{5}P_{0,t} / {}_{30}P_{15,t}) \times ({}_{30}P_{15,t+k}) \qquad [4]$$

where

P is the population,

t is the year of the most recent census, and

t+k is the estimation year.

In using the H-P method to forecast ${}_nL_x$ and in which the youngest age group is ${}_5L_0$ (ages 0-4, as is used in this paper), we obviously do not employ a CAR because the number of births in a life table is fixed, usually at 100,000 each year, which means that in a five year period (which corresponds to the width of abridged life table when 5 year age groups are employed up to the terminal, open-ended age group). Thus, ${}_5L_0$ is comprised of the survivors of these births. As such, one can simply take the ratio: ${}_5L_{0\ i,t}/{}_5L_{0\ i,t-k}$, or a variant thereof, as we do in this paper

Projections of the oldest open-ended age group differ slightly from the H-P projections for the age groups beyond age 10 up to the oldest open-ended age group. If, for example, the final closed age group is 80-84, with 85+ as the terminal open-ended age group, then calculations for the $CCR_{i,x+}$ require the summation of the three oldest age groups to get the population age 75+ at time t-k in a ten forecast cycle (80+ in a five year forecast cycle):

$${}_\infty CCR_{75,i,t} = {}_\infty P_{85,i,t} \ / \ {}_\infty P_{75,i,t-k} \qquad [5]$$

The formula for estimating the population of 85+ of area i for the year t+k is:

$${}_\infty P_{85,i,t+k} = ({}_\infty CCR_{75,i,t}) \times ({}_\infty P_{75,i,t}). \qquad [6]$$

An issue that is found in the cohort change ratio for the terminal, open-ended age group (which in our case is 85 years and over) in a projection where migration is not a component of population change is that like the equivalent probability of survival in an abridged life table, deaths are not uniformly distributed within the interval (Chiang, 1984: Lahiri, 2018; Swanson, Bryan, and

Chow, 2020). This issue tends to exaggerate the length of life for those aged 85 and over in an abridged life table and in an H-P projection.

Before turning to the next section, please note that we used a "Trended CCR" model in this paper. It was selected because in a preliminary exploration (the details of which we do not report here) we examined two forms of the CCR model, one in which the initial CCRs were kept constant and the other in which the initial CCRs were trended to the CCRs five year beyond the launch year and found the latter not only to be more accurate but also have fewer constraint violations. The trended model is described in detail in the "Evaluation" section.

Constraints. There are three major constraints in forecasting (or backcasting) ${}_nL_x$. These constraints affect any approach designed to forecast ${}_nL_x$ that is based on the fundamental population equation. First, the births (and deaths) in a life table are typically fixed at 100,000 annually. Second, ${}_nL_x$ cannot exceed this constraint in that it is limited to n*100,000 in a fixed width age group of n years. Third, ${}_nL_{x+i} \leq {}_nL_x$. Because the CCR approach (as well as the CCM approach) does not recognize these constraints, one must take care to make sure the forecast does not violate them. We discuss this in more detail in the "Evaluation" section.

Transferring Uncertainty. In regard to transferring uncertainty to a CCR forecast of ${}_nL_x$, the approach we take here follows that of Swanson and Tayman (2025a, 2025b). It employs the ARIMA (Auto-Regressive Integrated Moving Average) time series method in conjunction with work by Espenshade and Tayman (1982), whereby we can transfer the uncertainty information found in the ARIMA method's forecast to the population forecast provided by the CCM approach.

Before moving on to a description of the Espenshade-Tayman approach, we first clarify our use of the term "confidence interval" in regard to forecast uncertainty. It is more common to use the term “forecast interval” or “prediction interval” in the context of forecasting because a “confidence interval,” strictly speaking, applies to a sample (Swanson & Tayman, 2014: 204). However, underlying the approach we describe herein is the concept of a “superpopulation,” which, as discussed later, describes a population that is but one sample of the infinity of populations that will result by chance from the same underlying social and economic cause systems (Deming and Stephan, 1941). The concept of viewing a forecast as a sample leads us to choose the term “confidence interval” rather than forecast interval or prediction interval.

The uncertainty intervals for the ${}_nL_x$ forecasts are based on ARIMA models that forecast the uncertainty of population density (total population/land area) for the same horizon years. We use "density" because the Espenshade-Tayman (1989) method for translating uncertainty information does so from an estimated "rate," which in this case is the "rate" of population density. Other denominators could be used in developing such a "rate," such as the ratio of population to housing units. However, using the land area as the denominator provides a virtually constant denominator over time, thereby reducing the effort in assembling the "rate" data. It also serves as a stabilizing element regarding the use of ARIMA in that it dampens the effect of short-term population fluctuations more effectively than, say, housing units, which also can fluctuate over time and are not always in concert with population fluctuations. As should be obvious, the data assembled to develop the ARIMA age-specific density forecasts should encompass the base data used to develop the ${}_nL_x$ forecasts themselves. The case study we present meets this condition in that the historical annual record of each of the ${}_nL_x$ values cover the period (1959-2024) in which both the ARIMA models for generating the age-specific (${}_nL_x$) density forecasts and the CCR ${}_nL_x$ forecasts themselves are based. The land area of Estonia is approximately 42,388 km^2.

The approach we take to generate uncertainty measures follows that of Swanson and Tayman (2025a, 2025b), which employs the ARIMA (Auto-Regressive Integrated Moving Average) time series method in conjunction with work by Espenshade and Tayman (1982), whereby we can transfer the uncertainty information found in the ARIMA method's forecast to the population forecast provided by the H-P approach. As described by Smith, Tayman, and Swanson (2001: 172-176), an ARIMA model attempts to uncover the stochastic processes that generate a historical data series. The mechanism of this stochastic process is described—based on the patterns observed in the data series—and that mechanism forms the basis for developing forecasts. At its heart, the ARIMA time series model is a regression-like forecast method. It was popularized by Box and Jenkins (1976) and has been used in analyzing and forecasting business, economic, and demographic variables. Examples of its use in demographic forecasting include McNown et al. (1995), Pflaumer (1992), Tayman, Smith, and Lin (2007), and Zakria and Muhammad (2009).

As Smith, Tayman, and Swanson (2001: 172-176) discuss, an ARIMA model attempts to uncover the stochastic processes that generate a historical data series. The mechanism of this stochastic process is described—based on the patterns observed in the data series—and that mechanism forms the basis for developing forecasts. As noted earlier, up to three processes can represent the stochastic mechanism: autoregression, differencing, and moving average.

The autoregressive process has a memory in the sense that it is based on the correlation of each value of a variable with all preceding values. The impact of earlier values is assumed to diminish exponentially over time. The number of preceding values explicitly incorporated into the model determines its "order." For example, in a first-order autoregressive process, the current value is only a function of the immediately preceding value. However, the immediately preceding value is also a function of the one before it, which is a function of the one before it, and so forth. Thus, all preceding values influence current values, albeit with a declining impact. In a second-order autoregressive process, the current value is explicitly a function of the two immediately preceding values; again, all preceding values have an indirect impact.

The differencing process creates a stationary time series (i.e., one with constant average and variance over time, which, in turn, implies there is no trend in the series). A stationary time series is essential for the construction of ARIMA models. When a time series is non-stationary, it can often be converted into a stationary time series by calculating differences between values. First differences are usually sufficient, but second differences are occasionally required (i.e., differences between differences). Logarithmic and square root transformations can also convert non-stationary variances to stationary variances. The moving average represents a "shock" to the system or an event with a substantial but short-lived impact on the time series pattern. This impact has a limited duration, and then time series trends return to normal. The order of the moving average process defines the number of time periods affected by the shock.

The most general ARIMA model is usually written as ARIMA (p, d, q), where p is the order of the autoregression, d is the degree of differencing, and q is the order of the moving average. (ARIMA models based on time intervals of less than one year may also require a seasonal component.) The first and most subjective step in developing an ARIMA model is to identify the values of p, d, and q. The d-value must be determined first because a stationary series is required to identify the autoregressive and moving average processes correctly. The value of d is the number of times one has to “difference” the series to achieve stationarity (usually 0 or 1, but occasionally 2 in data with non-linear growth). The p- and q-values are also relatively small (often 0, 1, or 2). The autocorrelation (ACF) and partial autocorrelation patterns are used to find the correct values for p and q. For example, a first-order autoregressive model

[ARIMA (1, 0, 0)] is characterized by an ACF that declines exponentially and quickly and a PACF with a significant spike only at lag 1. Once p, d, and q are determined, maximum likelihood procedures are used to estimate the parameters of the ARIMA model. The final step in the estimation process is model diagnosis. An adequate ARIMA model will have random residuals, no significant values in the ACF, and the smallest possible values for p, d, or q. After a successful diagnosis is completed, the ARIMA model is ready to use.

In closing this description of the ARIMA process, we note that there are alternatives, such as dynamic linear modeling (Sevestre and Trognon, 1996), but we employ ARIMA because of our experience with it and its widespread use.

In terms of our actual results, the patterns of the autocorrelation (ACF) and partial autocorrelation functions (PACF) were used to find the correct values for p and q (Brockwell and Davis, 2016: Chapter 3). Each of the ${}_nL_x$ ARIMAs model have random residuals and the smallest possible values for p, d, or q, as determined by the Portmanteau Test (Ljung and Box, 1979; NCSS, 2024). Using these criteria, each of the selected ${}_nL_x$ ARIMA models has been determined to be adequate. We note that there may be other versions that also are "adequate" and that further refinement of the selection process can be done (e.g., using the augmented Dickey-Fuller test (Dickey and Fuller, 1979) to identify the amount of differencing required to achieve a stationary time series). We used the ARIMA procedure found in the NCSS Statistical Software System (NCSS, 2024) for this set of tasks. After giving an example of how this approach works, again note that we use Estonian data from the Human Mortality Database to generate and evaluate ${}_nL_x$ forecasts.

Here is an example of this process using the 2050 world population projection result produced by Swanson and Tayman (2025b: 4-5).

Let P = projected world population (at time t_i)

Let D = forecasted world population density obtained from ARIMA at time t_i, and

Let A = land area of the world (131, 821, 645 square kilometers).

The 2050 ARIMA density forecast shows 73.02, 76.81, and 80.60 persons per square kilometer, respectively, for the land area of the world as a whole (95% Lower Limit of forecasted D, forecasted D, and 95% Upper Limit of forecasted D, respectively). The relative widths of the Lower and Upper Limits are -0.04938 and 0.04938, respectively. The 2050 world population projection found at IDB is 9.7 billion. Multiplying 9.75 billion by -0.04938 and adding this product to 9.75 billion yields 9.27 billion, the 95% Lower Limit, and adding the product 9.75 billion × 0.04938 to 9.7 billion yields 10.23 billion, the 95% Upper Limit of the 2050 world population forecast found at IDB. Putting it all together, we can state that one can be 95% certain that the 2050 world forecast found at IDB is between 9.27 billion and 10.23 billion.

Underlying the Espenshade-Tayman method is the idea that a sample is taken from a population of interest. In this case, the ARIMA results represent the sample, and the CCM forecasts represent the population. This interpretation is de rived from the idea of a "super-population" (Hartley and Sielken, 1975; Sampath, 2005; Swanson and Tayman (2012, pp. 32–33). This concept can be traced back to Deming and Stephan (1941), who observed that even a complete census, for scientific generalizations, describes a population that is but one of the infinity of populations that will result by chance from the same underlying social and economic cause systems. It is a theoretical concept that we use to simplify the application of statistical uncertainty to a population forecast that is considered a statistical model in this context. This approach is conceptually and mathematically different from the classical frequentist theory of finite population sampling

(Hartley and Sielken (1975)), but as pointed out by Ding, Li, and Miratrix (2017), in practical terms, these two approaches result in identical variance estimators. As such, we believe that this approach is on solid statistical ground. Before moving on, we also note that using the Espenshade-Tayman method (1982) here is not new. In addition to being employed by Espenshade and Tayman (1982), it has been used by Swanson (1989), Roe, Swanson, and Carlson (1992) and Swanson and Tayman (2025a, 2025b) in demographic applications.

Evaluation Using HMD $_{n}L_{x}$ Data for Estonia. The evaluations utilize annual $_{n}L_{x}$ data taken from the full HMD set for Estonia (1959 to 2024). The evaluations of the point and interval results were launched from 2000 using 2000/1995 CCRs trended to 2005/2000 CCRs. The model is shown in Table 1 while the point and interval evaluations are found in Tables 2 and 3.

(TABLE 1 ABOUT HERE)

The form of the "Trended CCR" model can be seen in Table 1. As noted earlier, it was selected because in a preliminary exploration we examined two forms of the CCR model, one in which the initial CCRs were kept constant and the other in which the initial CCRs were trended to the CCRs five year beyond the launch year and found the latter not only to be more accurate but also have fewer constraint violations. In examining this issue, we found that using a weighted average between $_{5}L_{0}$ found at the point prior to the appearance of $_{5}L_{0} \geq 500{,}000$, with at least 80 percent of the weight on $_{5}L_{0}$ at the prior point and the remaining percent on $_{5}L_{0}$ at the point of initial appearance effectively eliminated this violation through the end of the 20 year forecast horizon we employed. We did not encounter any violations of the third constraint once we eliminated the initial appearance of the violation of the second constraint.

In regard to the violations found in the trended model, the first violation occurred in 2015 for ${}_5L_0$ (500,248), which was then adjusted to 499,243 (where 499,243 = .8*${}_5L_0$ in 2010 (498,573) + .2*${}_5L_0$ in 2015 (500,248)). The second violation occurred in 2020 when ${}_5L_0$ = 500,548 , which was adjusted to 499,243 (where 499,243 = .8*adjusted ${}_5L_0$ in 2015 (499,243 + .2*${}_5L_0$ in 2020 (500,584)).

(TABLE 2 ABOUT HERE)

In addition to showing the numeric and relative differences between the forecasted e_0 and the reported e_0, the summary measures shown in Table 2 are MALPE (Mean Algebraic Percent Error), MAPE (Mean Absolute Percent Error), and the Index of Dissimilarity Index (ID, also known as the Index of Misallocation, IOM). MALPE provides a view of bias in that if it is negative, then, on average, the forecasted values are lower than the reported values while MAPE (Swanson and Tayman, 2012: 268-270) shows the mean percent difference between the forecasted and reported values regardless of whether or not the forecasts were too high or too low. ID measures the extent that the forecasted values by age differ from the reported values by age. It is interpreted as the percent of the forecasted values by age that would have to be re-distributed in order to match the reported values by age (Swanson and Tayman, 2012: 273). In assessing these measures of error, we use guidelines found in Smith, Tayman, and Swanson (2013: 348-352) and Swanson and Tayman, (2012: 281-286) and define substantive errors as at least +-5% but less than +-10% and extreme errors (outliers) as being +-10% or more.

All of the MALPE and MAPE values are well below 5%. However, the ID measures arrange from a low of 9% in 2010 to a high of 12.05 % in 2020. Extreme errors (outliers) are neither summarized in Table 2 nor shown elsewhere but we can report that there are only three among the 18 age groups across the three evaluation points. All of them occur for age 85+. In

2010, the highest relative error is found for ${}_5L_{85}$ (-10.99%); in 2015, the highest error is again for ${}_\infty L_{85}$ (-9.9%); and in 2020, it is also for ${}_\infty L_{85}$ (-20.6%). These errors indicate that the method is more likely to have an error among the older age groups than the younger age groups. These errors are consistent with both the MALPE values and the differences in e_0, which are all negative. As we subsequently discuss, this suggests that over the period of time employed in the ex post facto evaluation portion of this case study of Estonia, the method over-estimates mortality on average in that it under-estimates ${}_nL_x$ on average.

With the exception of ID (where the 2015 values is higher than the 2020 value), MALPE, MAPE and both the absolute and relative difference between the forecasted e_0 and reported e_0 increase over time. With the exception of the 2015 ID value, this is consistent with the expectation that both uncertainty and errors are expected to increase over time as one moves farther away from the forecast launch year (Swanson and Tayman, 2025a, 2025b).

(TABLE 3 ABOUT HERE)

As seen in tables 3a.1, 3b.1, 3c.1 both of the two constraints that relate to the age groups are satisfied for all age groups in the 2010, 2015 and 2020 forecasts. As found in tables 3a.2, 3b.2 and 3c.2, the 66% confidence intervals encompass the reported 2010, 2015, and 2020 ${}_nL_x$ values, respectively, 100%, 94.4%, 88.9% of the time. In the 2015 forecast, the one occasion it does not is for age 85+ and in the 2020 forecast it does not encompass the reported values for 10-14 and 85+. These results are consistent with guidelines found in Swanson and Tayman (2014), wherein t 66% CIs should encompass the actual (reported) value at least 66% of the time.

Continuing with the interval estimates, the 66% “half-widths” ((UL66% - LL66%)/2) increase over time as should be the case as we expect uncertainty to increase as one goes further into the future. In 2010, the mean half-width is 24,297; in 2015, it is 30,668, and in 2020, it is 36,385. Accordingly, the mean half-width in the 2029, 2034, and 2039 forecasts (Tables 6, 7, and 8) are, respectively 48,067, 52,214, and 56,852. Accordingly, the intervals for e_0 also become wider over time. Their half-widths for 2010, 2015 and 2020 are, respectively, 4.37, 5.52, and 6.55.

The 66% confidence intervals generated for T_0 and e_0 (which recall is equal to T_0/100,000) started with the ${}_nL_x$ forecasts. There are two ways in which the ${}_nL_x$ confidence intervals can be used to place confidence intervals around a given T_0 (and subsequently for e_0 by simply dividing the lower limit (LL) and upper limit (UL) found for T_0 by 100,000), one is informal while the other is formal (Swanson and Tayman, 2014). In the informal approach, one would obtain confidence intervals for T_0 by adding, respectively, the LLs and ULs found for the ${}_nL_x$ values (i.e., the sum of ${}_nL_x$ LLs = T_0 LL and the sum of ${}_nL_x$ ULs =T_0 UL). The formal approach is called the "error propagation method" by Deming (1950: 127- 134). In different forms it has been used by Alho and Spencer (2005), and Espenshade and Tayman (1982), among others. This approach involves summing the squared values of the forecasted intervals, finding the square root of the summed forecast interval values and dividing this by the square root of the sample size to obtain an estimate of the standard error for the total forecast. This standard error is then multiplied by the total forecast (found by summing the point forecasts) to obtain the margin of error. The margin of error is added to and subtracted from the total forecast to obtain the interval associated with the desired level of confidence (66%, 95%, 99%). Applied to a forecast of T_0, this approach assumes that the ${}_nL_x$ forecasts are independent, which is not an unreasonable assumption in that they are not forced to sum to any specified total (i.e., they are not "controlled" to an externally produced T_0) and each ARIMA-based forecast of "${}_nL_x$ density" is a separate model. Swanson and Tayman (2014) report that both the informal and formal approaches generated virtually indistinguishable confidence intervals when aggregated from the "bottom-up" forecasts to the total forecast. As such, we employed the informal approach here.

Keeping in mind that Swanson and Tayman (2012: 275) point out that it is not generally possible to produce a population estimate for which all error criteria are simultaneously minimized, we find that the evaluation suggests that the method is slightly biased toward under-estimation of $_nL_x$ but is capable of producing point and interval forecasts that are sufficiently accurate that the method should be considered for use. This is with the proviso that evaluations of its performance should also continue, both in terms of populations that have mortality patterns similar to Estonia's over the case study period and in terms of population that have different mortality patterns. We conclude with the fact that to some extent, the COVID-19 pandemic (approximately January 2020 to May, 2023) to may have affected some of our results Rigby and Satija (2023).

An Example Forecast for Estonia. Because the evaluation data cover a 20 year forecast horizon from the launch year of 2000 (with the launch using 2000/1995 CCRs trended to 2005/2000 CCRs to forecast $_nL_x$ for 2010, 2015, and 2020) to the target year of 2020, we use the same horizon for the example forecast, which is launched from the most recent data (2024) available in the Human Mortality Database. The result is a forecast launched from 2019 (using 2019/2014 CCRs trended to 2024/2019 CCRs to forecast $_nL_x$ for 2029, 2034, and 2039) to the target year of 2039. This model is found in Table 4. The probabilistic $_nL_x$ forecasts it generates for 2029, 2034, and 2039 are found in tables 5, 6, and 7, respectively.

(TABLES 4, 5, 6, AND 7 ABOUT HERE)

In comparing the 66% $_nL_x$ and e_0 confidence intervals from 2029 to 2039, we find that on average their half-widths (calculated for ages 0-4 to 85+) increase over time as was the case in regard to the half-widths described in the preceding “Evaluation” section. In terms of the ${}_nL_x$ half-widths, they are for 2029, 2034 and 2039, respectively, 48,067, 53,214 and 56,852; in terms of the half-widths for e_0, they are, respectively, 8.65, 9.40, and 10.23. As was the case with the point and interval evaluations this is encouraging in that as time moves forward we expect uncertainty to increase. No violations of constraints 2 and 3 were found.

There are interesting differences between the forecasts of e_0 found, on the one hand in the evaluation period, 2010, 2015, ad 2020 , which were launched from 2000 and, on the other, in the forecasts for 2029, 2034, and 2039, which were launched from 2024. In the forecasts for 2010, 2015, and 2020 e_0 is, respectively, 74.71, 76.08, and 76.72; while in 2029, 2034, and 2039 they are, respectively, 80.60, 79.98, and 79.36.Thus, they increase until 2029 and then show slight declines from there to 2034 and 2039. In examining years lived and years remaining by age (15, 30, 45, 65, and 75) over the period 2010 to 2039, we find that years lived increased at all ages (15, 30, 45, 65, and 75) from 2010 to 2020 as did years remaining. Between 2020 and 2029, years lived decreased at age 15 and age 30, while for 45, 65, and 75, they increased; Between 2029 and 2039, we found that both years lived and years remaining decreased at all ages (15, 30, 45, 65, and 75).

It may be the case that the slight declines represent the possibility that Estonia’s population is bumping up against the expiration period of the “biological warranty” (Olshansky and Carnes, 2009). That is, Estonia’s population is coming up against the limits of human longevity. This interpretation is consistent with those on the side of the longevity debate who argue that continued increases in human longevity are not likely (see, e.g., Olshansky and Carnes, 2009; Swanson and Sanford, 2012) as opposed to those on the other side who argue that we can expect continued increases (see, e.g., de Gray, 2002; Kurzweil, 2005; Oeppen and Vaupel, 2002).

Discussion. As observed by Tóth (2021: 129), the efficiency of a given mortality forecasting approach largely depends on the character of the given time series, which explains variation in the usefulness of models with different demographic backgrounds. This observation applies not only to Estonia but to all of the other ${}_nL_x$ data sets found in HMD, which represent 41 countries deemed to have high quality mortality data. As a member of the UN's "Europe" region of the world, Estonia can be viewed as a sample of this region, one that is representative in terms of low fertility and low mortality, but not so much in terms of low migration. Given our ex post facto evaluation and its small population, our results suggest that the method will likely work in countries of a similar size as well as with larger populations that generally share its characteristics. It is an open question whether it will work in countries that have different demographic backgrounds. Here, again we note that the COVID-19 pandemic (approximately January 2020 to May, 2023) to may have affected some of our results Rigby and Satija (2023).

Constructing CCRs from two consecutive period life tables implies that the two life tables also represent cohort mortality. For example, in the 2000 and 1995 period life tables used to construct the CCRs for the evaluation, ${}_5L_{24}$ in 2020 is viewed as the cohort that five years earlier was five years younger, ${}_5L_{20}$. Given this, none of the CCRs beyond ${}_5L_0$ should exceed 1.00. However, as can be seen in Table 1, the CCR for these two age groups is 1.005507527, as do the rest of the CCRs from ${}_5L_5$ to ${}_5L_{45}$. Because period life tables are not constructed with cohorts in mind, these "anomalies" can occur when two successive period life tables are viewed in terms of sets of cohorts. This serves to remind us that the CCRs generated from two successive period life tables, which is the case in this approach to forecasting ${}_nL_x$, the CCRs represent approximations of the mortality experience of different sets of cohorts (e.g., in the period life table at time = t+5, ${}_5L_{10}$ is part of the cohort ${}_5L_0$ found at time = t-5 as is ${}_5L_5$ found in the period life table at time = t;

whereas in the period life table at time = t+5, $_5L_{15}$ is part of the cohort $_5L_0$ found at time = t-10 as is $_5L_{10}$ found in the period life table at time = t; and so on). If the approximation are close, such that the entire set of CCR approximates the mortality experience of these sets of cohorts, as apparently is the case with the CCRs in Table, the approach should work reasonably well over a 20 year period, as our evaluation indicates. If they do not, one can expect more violations of the constraints, which would require more adjustments than we needed in order to work or lead one to the decision not to use this approach if the violations are extensive and pronounced.

Thus, in regard to the constraints and the simple adjustment we used to overcome violations of these constraints, it may be the case that they may not work as well in other populations, especially those with different demographic backgrounds. As suggested earlier, a useful starting point for the resolution of violations is found in the "floors and ceilings" discussion found in Swanson Schlottman, and Schmidt (2010). And, of course, there are the many related tools found online that can be used for the purpose of overcoming these violations, such as those found at *DemoTools* (https://timriffe.github.io/DemoTools/index.html) and the *Applied Demography Toolbox* (https://applieddemogtoolbox.github.io/).

Unlike the Lee-Carter Method (and its variants) our new method is directly linked to the fundamental demographic equation, the cornerstone of demographic theory, an important consideration in developing accurate forecasts (Swanson et al, 2024). Because it forecasts "years lived," this new approach directly yields life expectancy (by age) via the summation of $_nL_x$ values from 85+ back to age group 0-4. This is not the case with the Lee-Carter Method and its variants, which would require life table construction from the forecasted ASDR's (e.g., Fergany's method (Fergany, 1971) and the Keyfitz-Frauenthal method (Kintner, 2004: 314-315)).

From the perspective of formal demography the CCR approach to forecasting ${}_nL_x$ is a means of forecasting the age structure and size of the stationary population that is associated with a given population (at a given point in time). As such, it can be viewed as a contribution to formal demography similar to contributions that demonstrated the CCR approach can used to take a given population to stability (Swanson, 2024; Swanson, Baker, and Tedrow, 2016). Viewed in this light, the expectation underlying such a forecast is that variance in age at death will continue even if those who argue that we can expect substantial improvements that lead to higher longevity levels (e.g., de Gray, 2002; Kurzweil, 2005; Oeppen and Vaupel, 2002). As pointed out by Swanson and Tedrow (2021), in order to have zero variance in age at death, all of the members of each birth cohort would have to die at the same time, which is so unlikely as to be impossible in most if not all species, including humans. Thus, given the other qualifications we have discussed, one could expect that the CCR approach to forecasting ${}_nL_x$ would work reasonably well in the face of dramatic improvements in human longevity.

Some may argue that the use of a simple forecasting method such as which we employ here lacks "real world" predictive ability. To such an argument we reply that Green and Armstrong (2015) find that while no evidence shows complexity improves accuracy, complexity remains popular among (1) researchers because they are rewarded for publishing in highly ranked journals, which favor complexity; (2) methodologists, because complex methods can be used to provide information that supports decision makers' plans; and (3) clients, who may be reassured by incomprehensibility. In regard to our simple forecasting method being "extrapolative," we note that virtually all "objective" forecasting methods not only include elements of judgement, but are in essence extrapolative and based on historical data, to include ARIMA (Box and Jenkins, 1976; Pflaumer, 1992), the Cohort Component Method (Smith, Tayman, and Swanson, 2013: 45-50);

structural models (Smith, Tayman, and Swanson, 2013: 215-238), the Lee-Carter mortality forecasting method (Lee and Carter, 1992; Basellini, Camarda, and Booth, 2023), and even what many would consider to be a "subjective" method – The Delphi Technique (Dalkey, 1969). Moreover, while forecasting comes with uncertainty, as Anatole Romaniuc (2010: 14) observed, "Uncertainty should not be a deterrent to exploring the future."

In terms of future research, it would be useful to conduct the same type of evaluation for different countries. In terms of the Human Mortality Database, this could be done by region of the world (as specified by the United Nations, there are five, Africa, Americas, Asia, Europe, and Oceania). For example, Australia (a member of the UN's "Oceania" region of the world), Canada (a member of the UN's "Americas" region of the world, and Japan, a member of the UN's "Asia" region of the world. Countries in these same regions not found in HMD may be found in the Human Life-Table Database (HLD) In terms of either the Human Mortality Database (41 countries) or the Human Life-Table Database (142 countries) this could be done by region of the world (as specified by the United Nations, there are five, Africa, Americas, Asia, Europe, and Oceania). Among HLD's 142 countries, there is a fair contingent from Africa, to include among others, Botswana, Cameroon, Egypt, Gambia, Ghana, South Africa, Tanzania and Zambia. Examples for other UN regions found in HLD regions include Indonesia, a member of the UN's "Oceania" region of the world, Argentina, a member of the UN's "Americas" region of the world, and India, a member of the UN's "Asia" region of the world.

In addition to further examination of the issues underlying the constraint violations, another area for future research is to conduct evaluations similar to those employed here in terms of ${}_nL_x$ by gender. Most life tables are by gender and this would be a natural area for the next step in future research.

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Table 1. The Estonian CCR Model using 2000/1995 CCRs Trended to 2005/2000 CCRs

AGE	nLx 1995	nLx 2000	nLx 2005	2000/1995 CCR	2005/2000 CCR	TREND IN CCR
0-4	491,905	495,239	496,903	1.006777731	1.003359994	0.996605271
5-9	489,931	493,945	495,991	1.004147142	1.001518459	0.997382173
10-14	488,919	493,226	495,393	1.006725437	1.0029315	0.996231409
15-19	487,197	492,183	494,501	1.006675952	1.002585022	0.995936199
20-24	483,157	489,316	492,104	1.00434937	0.999839491	0.995509651
25-29	478,144	485,818	488,625	1.005507527	0.998587825	0.9931182
30-34	471,701	481,540	484,818	1.007102463	0.997941616	0.990903759
35-39	462,750	475,408	480,016	1.007858792	0.996835154	0.989062319
40-44	448,901	465,412	472,441	1.005752566	0.993759045	0.988075078
45-49	430,629	449,868	460,489	1.00215415	0.989422275	0.987295493
50-54	407,835	429,409	441,805	0.997166935	0.982076965	0.984867158
55-59	375,809	403,415	417,639	0.989162284	0.972590234	0.98324638
60-64	339,481	370,022	387,411	0.98460122	0.960328694	0.975347861
65-69	294,444	328,973	348,749	0.969046869	0.942508824	0.972614281
70-74	244,418	276,452	300,849	0.938895002	0.914509701	0.974027659
75-79	185,599	213,398	240,189	0.87308627	0.868827138	0.995121751
80-84	119,846	144,657	169,435	0.779406139	0.793985886	1.018706225
85+	81,056	113,494	135,368	0.564922201	0.524375269	0.928225635
T_0	6,088,764	6,323,129	7,302,726			
e_0	60.89	63.23	73.03			

Table 2. Summary Measures of Error: Ex Post Facto Forecasts Launched from 2000.

YEAR	MALPE	MAPE	INDEX OF DISSIMILARITY
2010	-2.21%	2.25%	9.00%
2015	-3.24%	3.26%	13.21%
2020	-3.30%	3.31%	12.05%

Table 3a.1. 66% Confidence Intervals around the 2010 Trended CCR $_nL_x$ Forecast

AGE	66%LL	FORECAST	66%UL
0-4	497,743	498,573	499,402
5-9	496,728	497,658	499,402
10-14	496,497	497,445	498,393
15-19	494,943	496,674	498,404
20-24	491,107	494,422	497,736
25-29	483,125	491,409	499,630
30-34	479,399	487,619	495,840
35-39	470,618	483,284	495,949
40-44	457,656	477,020	496,385
45-49	439,143	467,444	495,744
50-54	418,076	452,236	486,395
55-59	386,870	429,695	472,521
60-64	338,586	401,071	463,555
65-69	312,879	365,138	417,398
70-74	268,490	318,934	369,379
75-79	218,461	261,386	304,311
80-84	155,984	190,707	225,429
85+	127,273	159,831	192,389
T_0	7,033,577	7,470,544	7,908,261
e_0	70.34	74.71	79.08

Table 3a.2 Number of Times the Forecasted 66% Confidence Interval Encompasses the Reported $_nL_x$ Value for Estonia, 2010

		REPORTED nLx
AGE	REPORTED	within 66% CI?
0-4	497,993	1
5-9	497,286	1
10-14	496,900	1
15-19	496,188	1
20-24	494,662	1
25-29	492,099	1
30-34	488,644	1
35-39	484,656	1
40-44	479,759	1
45-49	471,944	1
50-54	459,493	1
55-59	441,346	1
60-64	414,826	1
65-69	379,498	1
70-74	335,629	1
75-79	276,177	1
80-84	201,286	1
85+	179,561	1
T_0	7,587,947	
e_0	75.88	
		100.00%

Table 3b.1. 66% Confidence Intervals around the 2015 Trended CCR $_nL_x$ Forecast

AGE	66%LL	FORECAST	66%UL
0-4	496,287	498,908	499,280
5-9	496,594	497,992	498,464
10-14	496,359	497,779	498,274
15-19	494,967	497,395	498,898
20-24	490,904	495,264	498,702
25-29	481,992	492,401	501,827
30-34	478,745	489,084	498,513
35-39	468,990	484,774	499,657
40-44	454,976	478,981	502,096
45-49	435,584	470,710	504,961
50-54	415,144	457,836	499,677
55-59	384,588	438,662	491,919
60-64	332,597	411,543	489,724
65-69	309,914	377,000	443,385
70-74	267,736	333,028	397,702
75-79	220,838	276,356	331,361
80-84	161,900	206,981	251,677
85+	140,126	183,321	226,175
T_0	7,028,242	7,588,013	8,132,291
e_0	70.28	75.88	81.32

Table 3b.2 Number of Times the Forecasted 66% Confidence Interval Encompasses the Reported $_nL_x$ Value for Estonia, 2015

		REPORTED nLx
AGE	REPORTED	within 66% CI?
0-4	498,628	1
5-9	498,098	1
10-14	497,822	1
15-19	496,981	1
20-24	495,693	1
25-29	494,185	1
30-34	491,115	1
35-39	487,214	1
40-44	482,740	1
45-49	476,040	1
50-54	466,094	1
55-59	450,436	1
60-64	427,064	1
65-69	394,841	1
70-74	352,823	1
75-79	298,818	1
80-84	229,295	1
85+	228,595	0
T_0	7,766,482	
e_0	77.66	
		94.44%

Table 3c.1. 66% Confidence Intervals around the 2020 Trended CCR $_nL_x$ Forecast

AGE	66%LL	FORECAST	66%UL
0-4	497,935	499,243	499,623
5-9	496,922	498,327	499,623
10-14	496,555	498,114	498,616
15-19	495,093	497,729	499,440
20-24	491,116	495,983	499,927
25-29	481,406	493,239	504,083
30-34	478,313	490,071	500,917
35-39	468,159	486,230	503,397
40-44	452,864	480,458	507,158
45-49	432,055	472,645	512,355
50-54	411,364	461,035	509,849
55-59	379,574	444,094	506,798
60-64	326,562	420,131	512,919
65-69	306,301	386,844	466,668
70-74	265,128	343,847	421,926
75-79	221,664	288,569	354,936
80-84	164,958	218,835	272,306
85+	151,289	204,116	256,564
T_0	7,017,261	7,679,507	8,327,105
e_0	70.17	76.80	83.27

Table 3c.2 Number of Times the Forecasted 66% Confidence Interval Encompasses the Reported $_nL_x$ Value for Estonia, 2020

		REPORTED nLx
AGE	REPORTED	within 66% CI?
0-4	499,197	1
5-9	498,899	1
10-14	498,649	0
15-19	498,136	1
20-24	497,255	1
25-29	496,047	1
30-34	494,502	1
35-39	491,704	1
40-44	487,561	1
45-49	481,176	1
50-54	471,305	1
55-59	455,676	1
60-64	433,059	1
65-69	401,131	1
70-74	360,633	1
75-79	309,403	1
80-84	242,884	1
85+	256,986	0
T_0	7,874,203	
e_0	78.74	
		88.89%

Table 4. The Estonian CCR Model using 2019/2014 CCRs Trended to 2024/2019 CCRs

	${}_nL_x$	${}_nL_x$	${}_nL_x$	2019/2014	2024/2019	TREND
AGE	2014	2019	2024	CCR	CCR	IN CCR
0-4	498,506	499,109	499,245	1.001209614	1.000272486	0.999064003
5-9	497,930	498,656	498,934	1.000300899	0.999649375	0.999348672
10-14	497,648	498,391	498,638	1.000925833	0.999963903	0.99903896
15-19	496,901	497,917	497,954	1.000540543	0.999123178	0.998583401
20-24	495,135	496,825	496,715	0.999847052	0.997585943	0.997738545
25-29	492,941	495,378	495,616	1.000490775	0.997566548	0.997077207
30-34	489,674	493,327	493,933	1.000783055	0.997083036	0.996302875
35-39	485,770	490,713	491,130	1.00212182	0.995546564	0.993438667
40-44	481,204	486,987	487,023	1.002505301	0.99248033	0.990000082
45-49	474,364	480,718	480,855	0.998990033	0.987408288	0.988406546
50-54	463,299	471,203	471,521	0.993336341	0.980868201	0.98744822
55-59	447,155	456,335	458,462	0.98496867	0.972960698	0.987808778
60-64	422,570	435,470	438,654	0.973868122	0.961254342	0.987047753
65-69	389,250	404,363	409,513	0.956913647	0.940393138	0.982735632
70-74	346,909	363,401	371,092	0.933592807	0.917719969	0.982998115
75-79	292,709	310,232	319,331	0.894274867	0.878729007	0.982616239
80-84	223,280	242,813	252,343	0.829537185	0.813400939	0.980547892
85+	215,087	256,712	283,053	0.585609774	0.566644312	0.967614164
T_0	7,710,332	7,878,550	7,944,012			
e_0	77.10	78.79	79.44			

Table 5. Probabilistic 2029 ${}_nL_x$ Forecast

AGE	66%LL	FORECAST	66%UL
0-4	497,066	498,914	498,776
5-9	496,802	498,595	498,703
10-14	496,607	497,975	498,566
15-19	494,280	497,226	499,271
20-24	489,469	495,704	499,801
25-29	478,376	493,816	505,282
30-34	475,593	491,958	502,420
35-39	463,450	487,473	505,383
40-44	444,412	481,357	508,993
45-49	418,802	475,796	514,345
50-54	361,375	468,860	511,410
55-59	361,375	460,094	513,252
60-64	304,040	446,663	536,427
65-69	304,040	423,639	496,684
70-74	261,395	395,706	466,490
75-79	231,631	358,302	411,438
80-84	187,809	307,029	338,573
85+	236,655	280,748	427,767
T_0	7,003,176	8,059,854	8,733,579
e_0	70.03	80.60	87.34

Table 6. Probabilistic 2034 $_nL_x$ Forecast

AGE	66%LL	FORECAST	66%UL
0-4	497,059	498,582	498,783
5-9	496,797	498,414	498,708
10-14	496,600	498,098	498,573
15-19	494,120	496,834	499,431
20-24	489,093	494,904	500,177
25-29	477,338	493,053	506,315
30-34	474,560	490,555	503,453
35-39	461,798	486,554	507,034
40-44	441,808	478,969	511,597
45-49	414,807	469,785	518,340
50-54	354,405	460,835	516,533
55-59	354,405	450,621	520,222
60-64	291,527	436,539	548,939
65-69	291,527	412,787	506,961
70-74	251,229	382,172	476,655
75-79	223,829	341,674	419,241
80-84	182,864	285,774	343,518
85+	232,715	322,274	431,707
T_0	6,926,480	7,998,423	8,806,187
e_0	69.26	79.98	88.06

Table 7. Probabilistic 2039 $_nL_x$ Forecast

AGE	66%LL	FORECAST	66%UL
0-4	497,383	498,252	499,121
5-9	497,122	498,083	499,044
10-14	496,922	497,917	498,912
15-19	494,150	496,956	499,762
20-24	488,626	494,514	500,402
25-29	476,754	492,257	507,683
30-34	474,371	489,797	505,223
35-39	460,983	485,166	509,349
40-44	440,647	478,066	515,485
45-49	411,849	467,455	523,061
50-54	352,201	455,014	523,967
55-59	352,201	442,908	533,616
60-64	286,175	427,551	568,926
65-69	286,175	403,431	527,878
70-74	246,933	372,382	497,832
75-79	222,150	329,987	437,825
80-84	183,936	272,512	361,088
85+	230,346	333,389	436,431
T_0	6,898,921	7,935,635	8,945,603
e_0	68.99	79.36	89.46