

# GI 301 Model Solutions

## November 2025

### 1. Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

### Learning Outcomes:

- (1a) Identify the assumptions underlying the chain ladder estimation method.
- (1b) Test for the validity of these assumptions
- (1g) Assess whether a fitted model is acceptable given the data being modeled

### Sources:

*Outstanding Claims Reserves*, Hardy, SOA Study Note

### Commentary on Question:

*This item tested a candidate's ability to test for correlation when using the chain ladder method. The model solution is in the solutions spreadsheet. This document is for informational purposes only.*

### Solution:

- (a) Provide two reasons why the Spearman rank correlation is better.

### Commentary on Question:

*The solution below is an example of a full credit response.*

- It is more robust when there are outliers.
- It does not assume that the underlying random variables have the same variance.

- (b) Calculate the p-values using the two methods.

### Commentary on Question:

*The model solution is in the solutions spreadsheet.*

For both methods the  $DoF_j$  (degrees of freedom) for successive development stages  $(j-1, j)$  is the number of development factors in the vector for development stage  $x$  minus two [number of parameters = the factor at  $j-1$  and the factor at  $j$ ] minus one. For example, the correlation of development for  $(0,1)$  is  $9 - 2 - 1 = 6$ .

## 1. Continued

Pearson correlation factors,  $r_j$ , were calculated using the Excel function CORREL(development factor at  $j-1$ , development factor at  $j$ ). Note that the development factors at stage  $j-1$  is limited to the number of development factors at stage  $j$ .

Spearman correlation factors,  $r_j$ , were provided as values and did not require calculation.

The test statistic for correlation  $r_j$  is given by:

$$T_j = r_j \sqrt{\frac{n_j - 2}{1 - r_j^2}} = r_j \sqrt{\frac{DoF_j}{1 - r_j^2}}$$

Spearman correlation test statistics were provided as values and did not require calculation.

The p-value for  $T_j$  is from the two tailed Student's t-distribution. This is calculated using the Excel function T.DIST.2T(absolute value ( $T_j$ ),  $DoF_j$ ).

*(The solutions spreadsheet includes the formula used to calculate Spearman correlation and the formula used to calculate the test statistic for Spearman correlation. This is for informational purposes only).*

- (c) Evaluate whether the development factors are uncorrelated using the results in (b).

### **Commentary on Question:**

*The solution below is an example of a full credit response.*

None of the correlations between successive development factors are significant at any reasonable p-value using either Pearson or Spearman correlation (i.e., none of the p-values are under a reasonable threshold, such as 0.05). Therefore, the underlying assumption appears to hold for this data set.

## 2. Learning Objectives:

1. The candidate will understand how to use stochastic loss development models to estimate reserve variability.

### Learning Outcomes:

- (1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

### Sources:

*LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach*, Clark

*Stochastic Loss Reserving Using Generalized Linear Models*, Taylor, G., and McGuire, G., CAS Monograph Series Number 3,

### Commentary on Question:

*This item tested a candidate's understanding of Clark's parametric model for loss development. The model solution is in the solutions spreadsheet. This document is for informational purposes only.*

### Solution:

- (a) State the following:
  - (i) Clark's reasoning for statement I.
  - (ii) The observed random variables in Clark's approach.
  - (iii) The distribution that Clark assumes for the random variables in (ii).

### Commentary on Question:

*The solution below is an example of a full credit response.*

- (i) The Cape Cod method has many fewer parameters. With limited data points, the LDF method is likely prone to overfitting.
  - (ii) The random variables are the observed incremental losses.
  - (iii) The distribution is the over-dispersed Poisson distribution.
- (b) Estimate the process variance for the reserve, using Clark's approach and the information in tab "Q02".

### Commentary on Question:

*The model solution is in the solutions spreadsheet*

This is calculated using the information provided in cells R4:AA39 and O4:P11.

## 2. Continued

Sigma-hat squared is calculated as the sum of the following for rows 4 to 39:  
$$\frac{[\mu\text{-hat (column Z)} - \text{incremental paid (column U)}]^2}{\mu\text{-hat (column z)}}$$
all divided by the number of data points less the number of parameters (i.e.,  $36 - 3$ ).

Total reserve is [ELR (cell X43) times the total premium (sum of cells P4:P11)] minus total incremental paid (sum of cells U4:U39).

The standard deviation is then  $[\text{sigma-hat squared times the total reserve}]^{1/2}$ .

- (c) Your colleague stated that the semi-parametric bootstrap as described by Taylor and McGuire can be applied to Clark's model as an alternative way to estimate the variance of reserves.

Critique your colleague's statement.

**Commentary on Question:**

*There are many different approaches that could be used to formulate a full credit critique. The solution below is an example of a full credit solution.*

It is acceptable to apply the semi-parametric bootstrap to Clark's model. This bootstrap method is based on assuming that normalized residuals are identically and independently distributed. This is a reasonable assumption for Clark's model. Hence, bootstrap samples of residuals can be taken, and the variance of reserves calculated.

### 3. Learning Objectives:

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

#### Learning Outcomes:

- (2a) Estimate ultimate claims for excess limits and layers.
- (2b) Understand the differences in development patterns and trends for excess limits and layers.

#### Sources:

*Fundamentals of General Insurance Actuarial Analysis*, 2nd Ed. (2022), Friedland

- Appendix G

#### Commentary on Question:

*This question tested a candidate's knowledge regarding the development of excess limits and layers. An example of a full credit solution is in the solutions spreadsheet. This document is for informational purposes only.*

#### Solution:

- (a) Calculate total IBNR for claims excess of 500,000 limits using volume weighted average loss development factors.

#### Commentary on Question:

*The model solution is in the solutions spreadsheet.*

The question required loss development factors (LDFs) at each stage of development. However, loss triangle was missing a data point, losses for AY 2018 at 36 months of development. There are several reasonable approaches to handle a missing data point (e.g., ignore AY 2018 completely, ignore AY 2018 only for 24 to 36 and 36 to 48 LDFs, use a reasonable approach to create the value for the missing data point). The solutions spreadsheet uses the approach to ignore AY 2018 only for 24 to 36 and 36 to 48 LDFs.

An assumption for development after 84 months limited to the data provided is required. (e.g., judgement, Bondy method, exponential decay, log-linear regression, no further development). The solutions spreadsheet used the assumption of no further development after 84 months.

With the volume weighted LDFs, cumulative loss development factors (CDFs) are calculated. These are applied to the reported losses as of December 31, 2024 (i.e., the diagonal in the loss triangle) to calculate ultimate losses by AY. IBNR is then the ultimate losses minus the reported losses as of December 31, 2024.

### 3. Continued

- (b) Calculate total IBNR for the claims excess of 500,000 limits using Siewert's formula.

**Commentary on Question:**

*The model solution is in the solutions spreadsheet.*

Calculate the total limits CDFs from the age-to-age factors (i.e., LDFs) provided.

For claims excess 500,000, Siewert's formula for the CDF for excess of 500,000,  $CDF(x_{\$500,000})$ , at each stage of development is the total limits  $CDF_t \times (1 - R_{ULT}) / (1 - R_t)$ .

The IBNR for each AY is the AY reported losses excess of 500,000 as of December 31, 2024 times  $[CDF(x_{\$500,000}) - 1]$ .

- (c) Identify two reasons why alternative approaches should be considered for estimating the IBNR based on the results in (a) and (b).

**Commentary on Question:**

*The solution below is an example of a full credit solution.*

- There is a significant difference in the estimated IBNR between the two methods.
- The high CDF for report year 2024 indicates high volatility in the ultimate estimates.

- (d) Calculate IBNR for AYs 2023 and 2024 for the claims excess of 500,000 limits using the ILF method.

**Commentary on Question:**

*The model solution is in the solutions spreadsheet.*

Assume that the average loss occurrence date for AY 2023 is July 1, 2023, and for AY 2024 is July 1, 2024.

The AY 2023 trend period to January 1, 2024 is  $-0.5$  years. For AY 2024 it is  $0.5$  years. The large claim loading of  $1.46$  is then trended for each AY at the annual trend of  $5\%$ .

For each AY, the trended large claim loading is applied to the ultimate claims at basic limits to produce the ultimate claims at total limits.

### **3. Continued**

The IBNR for each AY for claims excess of 500,00 is the ultimate claims at total limits minus the ultimate claims at basic limits minus the reported claims excess of 500,000 limit (from the loss development triangle shown before part (a)).

#### 4. Learning Objectives:

4. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

#### Learning Outcomes:

- (4a) Describe a risk margin analysis framework.
- (4b) Identify the sources of uncertainty underlying an estimate of unpaid claims.
- (4c) Describe methods to assess this uncertainty

#### Sources:

*A Framework for Assessing Risk Margins, Marshall, et al.*

#### Commentary on Question:

*This question tested a candidate's understanding of risk margins as set out in Marshall. An example of a full credit solution is in the solutions spreadsheet. The solution in this file is for informational purposes only.*

#### Solution:

- (a) Identify one risk indicator from the list of potential risk indicators above for each risk component. Justify your selection.

#### Commentary on Question:

*The solution below is an example of a full credit response.*

Risk Component	Risk Indicator	Justification
Specification error	II	II relates to reducing the error that can arise from an inability to build a model that is fully representative of the underlying insurance process i.e., specification error
Parameter selection error	V	V relates to reducing the error that can arise because the model is unable to adequately measure all predictors of claim cost outcomes or trends i.e., parameter selection error.
Data error	III	III is clearly related to the error that can arise due to poor data or unavailability of data required. i.e. data error

- (b) Select a score of 2 or 4 for each risk indicator identified in (a).

#### Commentary on Question:

*Note that the spreadsheet cells D33:D35, for the risk indicator relating to the risk component, referenced column C in error as they should have referenced column D. Candidates were given full credit for the correct score applied to the risk component. For those candidates that did not correct this error, the risk indicator relating to the risk component in part (a) was used.*



## 4. Continued

*The solution below is an example of a full credit response.*

Risk Component	Risk Indicator	Score*
Specification error	II	4
Parameter selection error	V	4
Data error	III	2

- (c) Calculate the internal systemic risk CoV based on the completed balanced scorecard from (a) and (b) and the CoV scale provided.

**Commentary on Question:**

*The model solution is in the solutions spreadsheet.*

The weighted-average score is calculated as 3.4. Note that the weights are provided in the table before part (a) and the scores are from the response to part (b). Using the CoV scale with a score of 3.4 indicates a CoV of 9%.

- (d) Calculate the required risk margin for claim liabilities at a 75% probability of adequacy.

**Commentary on Question:**

*The model solution is in the solutions spreadsheet.*

IND = independent risk

ESR = external systemic risk

ISR = internal systemic risk

The margin is calculated as:

$z \times 750 \text{ million} \times (\text{CoV}(\text{IND})^2 + \text{CoV}(\text{ESR})^2 + \text{CoV}(\text{ISR})^2)^{0.5}$   
as IND, ESR and ISR are not correlated.

## 5. Learning Objectives:

5. The candidate will understand the methods to monitor actual versus expected experience.

### Learning Outcomes:

(5a) Identify and describe approaches for monitoring results.

(5b) Prepare a comparison of actual to expected claims.

### Sources:

*Fundamentals of General Insurance Actuarial Analysis*, Friedland, 2nd Ed. (2022)

- Chapter 37: Monitoring Results

### Commentary on Question:

*This question tested a candidate's ability to compare actual to expected results for an actuarial analysis. An example of a full credit solution is in the solutions spreadsheet. The solution in this file is for informational purposes only.*

### Solution:

Compare actual minus expected results as of March 31, 2025, for each of accident year 2021 to 2024, assuming ultimate claims are based upon each of the following:

- (i) Selected
- (ii) Reported development method

### Commentary on Question:

*The model solution is in the solutions spreadsheet.*

ULT = ultimate claims

LDF = loss development factor

CDF = cumulative development factor

RDM = reported development method

For both (i) and (ii), the following calculations are completed to arrive at actual minus expected amounts by AY. (Text in red indicates the cells in the solutions spreadsheet for this question.)

- 1)  $\text{Reported}_{\text{AY}}$  as of Dec. 31, 2024 (H28:H32) = paid + case as of Dec. 31, 2024
- 2)  $\text{Reported}_{\text{AY}}$  as of March 31, 2025 (I28:I32) = paid + case as of March 31, 2025)
- 3)  $\text{CDF}_{\text{AY}}$  (B36:B40, H36:H40) =  $\text{ULT}_{\text{AY}}$  [Selected ULT for (i), RDM for (ii)] /  $\text{Reported}_{\text{AY}}$

## 5. Continued

- 4)  $\text{LDF}_{\text{AY}} \text{ next 1 year } (C36:C40, I36:I40) = \text{Reported}_{\text{AY}} \text{ at age/ (b) Reported}_{\text{AY}} \text{ at (age -1year)}$
- 5)  $\text{LDF}_{\text{AY}} \text{ next 3 months } (D36:D40, J36:J40) = \text{LDF}_{\text{AY}} \text{ for next year } ^{0.25}$
- 6)  $\text{AY Expected as of March 31, 2025 } (E36:E40, K36:K40) = \text{LDF}_{\text{AY}} \text{ next 3 months } \times \text{Reported}_{\text{AY}}$
- 7)  $\text{AY Actual minus AY Expected } (F36:F40, L36:L40) =$   
 $[\text{Reported}_{\text{AY}} \text{ as of March 31, 2025} - \text{AY Expected first 3 months 2025}]$

*Note that a comparison can only be made for AYs 2021 to 2024.*

## 6. Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

### Learning Outcomes:

- (6c) Price for deductible options and increased limits

### Sources:

*Fundamentals of General Insurance Actuarial Analysis*, Friedland, 2nd Ed. (2022)

- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

### Commentary on Question:

*This question tested a candidate's understanding of deductibles and the coinsurance clause in an insurance contract. An example of a full credit solution is in the solutions spreadsheet. The solution in this file is for informational purposes only.*

### Solution:

- (a) Calculate the insured's retained loss for the following claim and insured amount scenarios.

(i) Claim amounts: 18,000, 21,500 and 25,000

(ii) Policy insured amounts: 16,000, 19,000, 22,000

### Commentary on Question:

*The model solution is in the solutions spreadsheet.*

For this question, to avoid the coinsurance penalty, the insured value must be at least 80% of the home value (20,000). When there is a coinsurance penalty, it is calculated as a percent using the formula  $1 - [\text{insured value} / (80\% \text{ of home value})]$ .

A 3 by 3 grid can be set up as insured value by loss amount for the insurer's claim amount. The amount in each cell is calculated as the minimum of (insured value, loss amount  $\times (1 - \text{coinsurance penalty \%})$ ) – deductible.

A similar 3 by 3 grid can then be set up for the insured's retained loss. The amount in each cell is the loss amount less the insurer's claim amount.

- (b) Calculate the amount the insurer would pay to the insured for the loss of their home.

### Commentary on Question:

*The model solution is in the solutions spreadsheet.*

## 6. Continued

The first step is to check if the coinsurance penalty applies. Here we have the insured value (200,000) less than 85% of the home value (212,500), so coinsurance would normally apply. However, the loss (250,000) is greater than the minimum insured value required by the coinsurance clause (212,500) so there is no coinsurance penalty. So, the insured loss is limited to the insured value of 200,000 without a coinsurance penalty. The EQ deductible applies to this loss. The insured loss is then 180,000 (200,000 – 10% of 200,000).

- (c) Explain what the insurer should expect regarding the frequency, severity and total claims if it made this change. State all assumptions.

### **Commentary on Question:**

*There are many different approaches to explaining the insurer's expectations. A full credit solution should consider both expectations if exposures remain stable and exposures are affected by the change. The following solution is an example of a full credit response.*

Assuming no change in written exposures:

- Increasing deductibles will reduce the frequency of claims, since claims between 2000 - 4000 would not be paid
- The severity change will depend on the distribution of exposures and the loss distribution. The total loss amount would be reduced.

However, the assumption of no change in exposures is not reasonable. A change such as this would likely have a significant impact on written exposures.

- Exposure changes will depend on the insurer's position in the market for premiums (high, competitive, low) and deductibles.
- Exposure changes will depend on the amount of premium reduction for the increased deductible.
- Customers that want a 2,000 deductible will see if it is offered by another insurer at an affordable premium.
- The insurer should expect exposure changes dependent on their pricing strategy and position in the market. This will affect both their premiums and losses.

## 7. Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

### Learning Outcomes:

- (6a) Understand and apply classification ratemaking methods
- (6b) Explain the issues and considerations regarding classification ratemaking.

### Sources:

*Fundamentals of General Insurance Actuarial Analysis*, Friedland, 2nd Ed. (2022)

- Chapter 33: Basic GI Risk Classification

### Commentary on Question:

*This question tested a candidate's understanding of risk classification relativities and how to calculate them using both a one-way analysis and the minimum bias method. An example of a full credit solution is in the solutions spreadsheet. The solution in this file is for informational purposes only.*

### Solution:

- (a) Apply a one-way analysis to each rating variable to determine their relativities.

#### Commentary on Question:

*The model solution is in the solutions spreadsheet.*

These calculations are straightforward and displayed in the solutions spreadsheet so that they are easy to follow. The first step is to calculate the pure premiums from the data provided.

The only issue in the calculation of the pure premium relativities is that relativities for the experience by classification category is to the total experience, not the base premium.

- (b) Assess whether the one-way analysis is suitable to determine the relativities for both rating variables.

#### Commentary on Question:

*There are several different approaches for assessing this. The key is to test for distributional bias as this would make a one-way analysis unsuitable. A full credit response should explicitly show a distributional bias in the exposures and make the proper conclusion. The model solution in the solutions spreadsheet is an example of a full credit solution.*

## 7. Continued

The solutions spreadsheet shows distributional bias by showing how the distribution by territory differs significantly by type of use.

- (c) Calculate the revised relativities for “Type of Use” that result from a single iteration of the minimum bias method.

**Commentary on Question:**

*The model solution is in the solutions spreadsheet.*

For full credit, the response was required to start with the one-way analysis relativities for the “Territory” rating variable (as indicated in the information provided for part (c)). These calculations are straightforward and displayed in the solutions spreadsheet so that they are easy to follow.

## 8. Learning Objectives:

6. The candidate will understand and be able to apply ratemaking techniques for the following situations: classification ratemaking, deductible options, increased limit options, claims-made policies and individual risk rating.

### Learning Outcomes:

- (6e) Understand and apply techniques for individual risk rating.

### Sources:

*Fundamentals of General Insurance Actuarial Analysis*, Friedland, 2nd Ed. (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

*Estimating the Premium Asset on Retrospectively Rated Policies*, Teng and Perkins, Casualty Actuarial Society

### Commentary on Question:

*This question tested a candidate's understanding of several issues regarding individual risk rating. An example of a full credit solution is in the solutions spreadsheet. The solution in this file is for informational purposes only.*

### Solution:

- (a) Compare retrospective rating to experience rating.

#### Commentary on Question:

*The solution below is an example of a full credit response.*

Experience rating is a prospective assessment of an insured's premium based on their past claims experience.

Retrospective rating is a retroactive adjustment to a deposit premium based on actual claims experience during the policy period.

- (b) Evaluate each company for retrospective rating, from the perspective of the insurance company.

#### Commentary on Question:

*There are several different approaches for evaluating this. The key is that it should be from the perspective of the insurance company. A full credit response should explain for each of A, B and C why or why not the insurer should use retrospective rating. The explanations should include numerical support. The model solution in the solutions spreadsheet is an example of a full credit response.*



## 8. Continued

The model solution in the solutions spreadsheet shows an example of some of the statistics (but not all) that could be shown as part of the numerical evaluation. Not all of these were required for full credit, however it was expected that something regarding the variability of annual results for each company be shown.

- (c) Recommend for each company the policy that should be selected. Justify your recommendation.

**Commentary on Question:**

*There are several different approaches for the recommendations. The key is that it should be from the perspective of the insured. A full credit solution should explain for each of A, B and C why they should accept the fixed premium policy or the retrospectively rated policy as offered by the insurance company. The justification should compare the fixed premium to the expected retrospectively rated premium. The model solution in the solutions spreadsheet is an example of a full credit response.*

The solution in the solutions spreadsheet shows an example of some of the statistics (but not all) that could be shown to justify the recommendations. The key was to compare the expected retro premium to the fixed premium for each company.

- (d) Compare the Berry approach to estimate the retro reserve to the approach used by Teng and Perkins.

**Commentary on Question:**

*Note that Berry was involved with two approaches, one using premium deviation (also used by Fitzgerald) and the other using premium development. Either of these approaches was acceptable for the comparison. The solution below is an example of a full credit response using the Berry premium development approach.*

Berry estimates ultimate premium using the historical premium emergence pattern then subtracts current premium. This is like the claims development method but with premiums. Teng & Perkins predict premiums using expected loss development and the retro-rating parameters, using premium development to loss development (PDLR) ratios. The PDLR ratios can be derived formulaically or based on experience.

## 9. Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts

### Learning Outcomes:

- (7d) Understand the issues encountered when applying loss development methods to reinsurance.

### Sources:

*General Insurance: Considerations in Reinsurance Reserving*, Cappelletti, SOA Study Note

### Commentary on Question:

*This item tested a candidate's knowledge of issues in reinsurance reserving. The model solution is in the solutions spreadsheet. This document is for informational purposes only.*

### Solution:

- (a) Calculate ceded ultimate claims for each accident year from PIC's perspective.

#### Commentary on Question:

*The model solution is in the solutions spreadsheet.*

This required calculating the net ultimate claims for each accident year using the reported development method on the net reported claims triangle. The gross ultimate claims were provided. The difference between the two is the insurer's ceded ultimate claims by accident year.

- (b) Calculate RB's loss ratio for each accident year.

#### Commentary on Question:

*The model solution is in the solutions spreadsheet.*

This required calculating the ceded ultimate claims for each accident year using the Bornhuetter Ferguson method on ceded reported claims with ceded CDFs, reinsurance premium (3% of insurer's gross earned premium) and the reinsurance ELR (60%). The ceded CDFs are calculated with the provided ceded LDFs.

- (c) State four reasons why a reinsurer's estimate of ultimate ceded claims may be different from a direct writer's estimate.

#### Commentary on Question:

*There are more than four reasons. The following solution is an example of a full credit response.*

## 9. Continued

- 1) Each entity may have different reserving philosophies (methodology, conservativeness).
- 2) The reinsurer would not generally have access to gross claims data, while the insurer would.
- 3) The reinsurer would base development patterns using amounts grouped from multiple ceding copies, whereas the insurer does not have this information.
- 4) The reinsurer may be subject to different legislation or financial reporting requirements that affect the standard for reserve estimates.

## 10. Learning Objectives:

7. The candidate will understand the fundamentals of reinsurance and demonstrate knowledge of reinsurance reserving, the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts

### Learning Outcomes:

- (7h) Understand the application of a reinstatement premium.

### Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm, Casualty Actuarial Society

### Commentary on Question:

*This question tested a candidate's understanding of determining the existence of sufficient risk transfer in a reinsurance contract to use (re)insurance accounting. An example of a full credit solution is in the solutions spreadsheet. The solution in this file is for informational purposes only*

### Solution:

- (a) Apply the following methods to determine whether this reinsurance contract transfers sufficient risk to permit reinsurance accounting using
  - (i) Expected Reinsurer Deficit (ERD) with a 1% threshold
  - (ii) The 10-10 rule

### Commentary on Question:

*The model solution is in the solutions spreadsheet.*

The ERD calculation requires the calculation of Gain / Loss on a present value (PV) basis considering premium, commission and claims.

- (b) Describe two advantages and one disadvantage of the ERD method over the 10-10 rule for assessing risk transfer.

### Commentary on Question:

*There are more than two advantages and more than one disadvantage. The solution below is an example of a full credit response.*

Advantage 1) The cutoff point for risk is economic breakeven, rather than a statistical percentile; Therefore, it could be considered more meaningful than a statistical definition based on a percentile, because the impact of risk on a company is in economic terms.

## 10. Continued

Advantage 2) It can identify risk transfer when there is a very small chance of catastrophic loss.

Disadvantage 1) It is more difficult to calculate.

(c)

- (i) Describe what RCR measures.
- (ii) State both versions of the formula for RCR (i.e., with and without ERD in the formula). Define all of the terms included in each formula.

**Commentary on Question:**

*The solution below is an example of a full credit response.*

$E[G]$  = expected economic gain across all possibilities,

$P$  = premium,

$p$  = probability of net economic loss, and

$T$  = average severity of net economic loss, when it occurs

$$RCR = E[G] / (pT)$$

and

$$RCR, \% \text{ form} = ERD / (E[G] / P)$$

## 11. Learning Objectives:

8. The candidate will understand catastrophe modeling output and the allocation of catastrophe risk loads among accounts.

### Learning Outcomes:

- (8a) Understand the purpose and development of catastrophe models.
- (8b) Understand the type of output produced by catastrophe models.
- (8c) Understand how catastrophe model output can be used in actuarial tasks.
- (8d) Allocate a property catastrophe risk load among different accounts.

### Sources:

*An Overview of Catastrophe Modeling Output*, Cappelletti, SOA Study Note

*Uses of Catastrophe Model Output*, American Academy of Actuaries

*An Application of Game Theory: Property Catastrophe Risk Load*, Mango, Casualty Actuarial Society

### Commentary on Question:

*This question tested a candidate's knowledge of catastrophe modeling output, and the allocation of risk loads to property catastrophe insurance accounts. The model solution is in the solutions spreadsheet. This document is for informational purposes only.*

### Solution:

- (a) State two advantages of using catastrophe models instead of historical data.

#### Commentary on Question:

*There are more than two advantages. The solution below is an example of a full credit response.*

Catastrophe models simulate significantly more realistically plausible events than those that may be contained in the historical record.

Catastrophe models allow users to analyze the current exposure and settlement terms, therefore avoiding the pitfalls in adjusting historical experience to reflect changes in insured properties exposed to the hazard and the insurance terms.

- (b)
- (i) Calculate the occurrence exceedance probabilities (OEP)
- (ii) Plot the OEP curve for Company X.

## 11. Continued

### **Commentary on Question:**

*The model solution is in the solutions spreadsheet.*

For part (i), events are sorted from lowest to highest probability events so no re-sorting is necessary. Calculate for each event (1)  $1 - \text{probability } (p_i)$ , and (2) the product of  $[1 - p_j]$  for  $j=1$  to  $i$ . The OEP is then 1 minus the amount from (2) for each event.

For part (ii), horizontal graph data is the amount of loss and vertical graph data is the OEP.

- (c) Calculate the renewal risk load for Company X and Y.

### **Commentary on Question:**

*The model solution is in the solutions spreadsheet.*

The events in the OEP table are given as independent Bernoulli events (This is because the OEP is the probability that the largest loss exceeds a specified amount). As such, the variance formula uses the sum from  $i=1$  to 10 of  $\text{OEP} \times (1 - \text{OEP}) \times \text{Loss}_i \times \text{Loss}_i$ .

The marginal variance for X is the variance of the total minus the variance of Y. The marginal variance for Y is the variance of the total minus the variance of X. The MV risk load is the MV risk load multiplier times the marginal variance.

- (d) Calculate the renewal risk load for Company X and Y using the Shapley Method.

### **Commentary on Question:**

*The model solution is in the solutions spreadsheet.*

The covariance of X and Y is  $[\text{Variance}(X+Y) - \text{Variance}(X) - \text{Variance}(Y)]/2$ .  
The Shapley value of X is  $\text{Variance}(X) + \text{covariance of X and Y}$ .  
The Shapley value of Y is  $\text{Variance}(Y) + \text{covariance of X and Y}$ .

The Shapley risk load for each account is the MV risk load multiplier times the Shapley value for each account.